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Processes of synchronization, chaotization and amplification in a germanium oscillistor

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Abstract

The effect of an external harmonic signal on the screw instability of the current in the electron-hole plasma has been studied experimentally in Ge at 77 K and 300 K. The influence exerted by external signals with various amplitudes and frequencies, applied to a sample both additively and multiplicatively, on the synchronization, amplification and stability of the system in absolute and convective modes of instability excitation has been investigated at points of bifurcation in a wide region of the parametric space.

1. Introduction

When subjected to external influence (electric and magnetic fields, illumination, temperature gradients, injection), semiconductors exhibit strong nonlinear behaviour leading to various kinds of instability, generation of oscillations and waves, occurrence of chaotic states, and spontaneous formation of spatial and temporal structures. Here, the results are reported of an experimental study of nonlinear dynamic processes associated with the evolution of the Kadomtsev–Nedospasov instability (screw instability or oscillistor effect) [1–4] in an injected electron–hole plasma in Ge in longitudinal electric and magnetic fields at high control parameters and temperatures of 77 and 300 K. The oscillistor was selected to study the synchronization and amplification processes because of the good reproducibility of experimental results and comparative simplicity of the experiment in this case. Samples of n-type germanium with equilibrium concentration $N_D - N_A \approx (10^{12}–10^{14})$ cm^{-3} were cut out as cylindrical or rectangular bars of length 10 mm and cross-section 1 mm^2 and etched in a polishing etchant. Contacts of In with 0.5% Ga and Sn with 7% Sb were deposited onto two opposite edges of the samples to provide an injection of, respectively, electrons and holes. Up to five pairs of 'Hall' probes allowing a study of the spatio-temporal coherence in the system were applied along the sample, with their ohmic behaviour in external electric fields verified. All the experiments were carried out in the steady-state mode, using rectangular pulses of varied duration and amplitude (figure 1).

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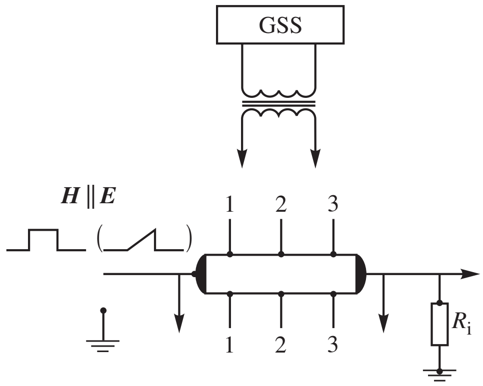

Figure 1. Schematic diagram of the experimental arrangement for a sample with three pairs of 'Hall' probes. A generator of sinusoidal signals (GSS) is connected to the sample by means of a pulse transformer.

2. Experimental results and discussion

The response of a nonlinear dynamic system in the state of self-oscillation mode or in the pre-threshold state to an external periodic perturbation depends on the frequency and amplitude of this perturbation [5] and also on the method by which it is introduced into the system. Depending on the frequency detuning $\Delta = \omega_{ex} - \omega_0$, where ω_0 is the fundamental frequency of a self-sustained oscillation system and ω_{ex} is the external frequency, various kinds of bifurcations and effects of weak signal amplification and frequency synchronization (or capture) have been found theoretically [5–7] and experimentally [8–10] in nonlinear dynamic systems at the fundamental frequency, harmonics and subharmonics of the external perturbation. An external perturbation $E_{ex} \sin(\omega_{ex} t)$ can be introduced into the system (i) additively, through opposite-end contacts of the sample, being superimposed on the dc electric field E_0 as an external force (force perturbation), with the total electric field given by

$$E = E_0 + E_{ex} \sin(\omega_{ex} t)$$

or (ii) multiplicatively, by means of a transformer through 1–1 potential probes (parametric perturbation), with the response of the system detected either at the 2–2 or 3–3 potential probes or across the resistance R_i.

2.1. Absolute instability mode

In the case of an absolute current instability (self-oscillation mode), the amplitude of oscillations arising at some point in space increases at this same point in the $t \to \infty$ limit until nonlinear effects restrict its build-up. In this case, voltages across the potential probes and the current through a sample show spontaneous oscillations.

If the control parameters E and H are chosen in such a way that the system is passing from unstable focus to limit cycle, then, on applying a small external harmonic signal to the sample, this signal is amplified. At large amplitudes of self-oscillations, when limit cycles occur in the system (figure 2), this signal is suppressed. Such a small perturbation results in different modulation patterns when the system passes from the limit cycle to, e.g., period-2 or period-4 bifurcations.

At $U_0 = 12$ V and $H = 4.5$ kOe, the sample is in the self-oscillation mode with a fundamental oscillation frequency $\omega_0 = 125\,724$ Hz (limit cycle). If an external periodic

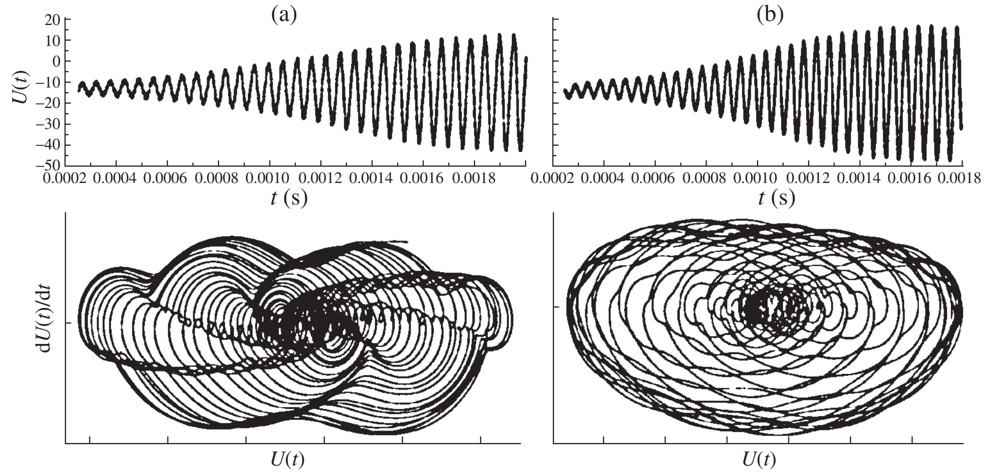

Figure 2. The effect of small external periodic perturbation on the system passing to the limit cycle: (*a*) $\omega_{ex} = 80\,490$ Hz and (*b*) $\omega_{ex} = 122\,451$ Hz.

perturbation with an amplitude $U_{ex} = 0.077$ V and frequencies $\omega_{ex} = \omega_0, \omega_0/2, 3\omega_0/2$ and $2\omega_0$ is additively applied to the sample, then frequency lock and 5–10-fold signal amplification are observed, with the strongest amplification occurring at frequencies ω_0 and $\omega_0/2$. By choosing $U_0 = 15.5$ V and $H = 4.5$ kOe, we bring the system into a threshold mode before the period-doubling bifurcation. With increasing U_0, a period-doubling bifurcation occurs, with the fundamental frequency ω_0 growing to 167 846 Hz and the fundamental harmonic exceeding the subharmonic $\omega_0/2 = 83\,923$ Hz in amplitude by a factor of 1.5. If an external signal with frequencies ω_0 and $\omega_0/2$ is introduced under these conditions, the harmonic with $\omega_0/2$ is amplified and the fundamental harmonic ω_0 decreases in intensity, in good agreement with the theory [6, 7]. Introduction of intermediate frequencies in the above-mentioned frequency detuning range gives rise to complex modulation patterns, manifesting themselves in power spectra as linear combinations of frequencies $n\omega_0 + \omega_{ex}$. In this case, two-dimensional tori with closed trajectories or with two incommensurate frequencies are observed in phase portraits, depending on the ratio of frequencies ω_0 and ω_{ex}. At $U_0 = 16.3$ V and $H = 4.5$ kOe, a period-4 bifurcation occurs and the fundamental harmonic frequency increases to $\omega_0 = 180\,053$ Hz, but peak power is observed at a frequency $\omega_0/2 = 90\,086$ Hz. Introduction of an external signal with frequencies ω_0, $\omega_0/2$ or $\omega_0/4$ results in a much smaller amplification and substantial distortion of trajectories in the phase space. A sample with screw instability developed upon introduction of an external periodic perturbation can be regarded as a Van der Pol–Duffing oscillator with external harmonic perturbation. All the characteristic features inherent in such an oscillator are manifested in the system under study. Theoretical investigations [11] predict for the transition from quasi-periodicity to chaotic state a universal behaviour with fractal dimension D. We calculated the dimension of the system by constructing the so-called Arnold tongues in the frequency synchronization range. Frequency synchronization occurs at frequency ratio

$$\frac{\omega_{ex}}{\omega_0} = \frac{p}{q}$$

where p and q are rational numbers, ω_0 is the fundamental frequency of self-oscillations, and ω_{ex} is external frequency.

It is known that the synchronization range is wider, the higher the amplitude of the external periodic signal. Let us denote by S the length of the interval between two ranges where the frequency locking occurs with ratios P_n/P_{n+1} and P_{n+1}/P_{n+2}, (P_n, P_{n+1} and P_{n+2} are numbers of the first Fibonacci sequence), and by S_1, S_2 the lengths of the intervals between the locked state $(P_n + P_{n+1}) / (P_{n+1} + P_{n+2})$ and states P_n/P_{n+1} and P_{n+1}/P_{n+2}, respectively. In the limit $n \to \infty$ the fractal dimension of the system can be estimated using the formula [11]

$$\left(\frac{S_1}{S}\right)^D + \left(\frac{S_2}{S}\right)^D \approx 1.$$

We obtained the frequency synchronization ranges corresponding to 3:5, 5:8 and 8:13 frequency locking ratios in the intervals 82.397–83.469 kHz, 89.517–90.112 kHz and 86.906–87.327 kHz, respectively. The fractal dimension at the onset of the chaotic state, estimated from our experiment, was $D = 0.903$, which is comparable with the theoretically calculated value of 0.868. The above-mentioned universal behaviour predicted by renormalization-group methods is observed when the winding number ρ determined experimentally as the ratio of ω_{ex} and ω_0 is maintained constant and equal to the golden mean $\sigma_g = (\sqrt{5} - 1)/2 = 0.618034\ldots$. Therefore, when raising the additional external field E_{ex} in experiment, we adjusted the external frequency ω_{ex} so as to ensure $\omega_{ex}/\omega_0 = \sigma_g$. Some complicating factors, such as fluctuations of the fundamental frequency ω_0 of the oscillator and instability of the frequency ω_{ex} of the generator of sinusoidal signals, may occur in experiment. Quasiperiodic transition to chaos is characterized by the universal exponent δ from the relation [11]:

$$\Omega_n(k) = \Omega_\infty(k) - \text{const}\, \delta^{-n}$$

where Ω_n is a parameter of the system, defining the winding number. The δ value is found from experiment as

$$\delta = \lim_{n\to\infty} \frac{\Theta_n - \Theta_{n+1}}{\Theta_{n+1} - \Theta_{n+2}}$$

where Θ_n, Θ_{n+1}, and Θ_{n+2} represent the widths of the locked states P_n/P_{n+1}, P_{n+1}/P_{n+2} and $(P_n + P_{n+1})/(P_{n+1} + P_{n+2})$ mentioned above. We measured the widths of 3:5, 5:8 and 8:13 locked states and obtained experimentally $\delta = 2.72\ldots$. Our value is in agreement, as an intermediate, with the theoretical values $2.618\ldots$ and $2.833\ldots$ for $|k| < 1$ and $|k| = 1$, respectively [11].

Using the time series obtained from different pairs of 'Hall' probes, we calculated the fractal and Kaplan–Yorke [12, 13] dimensions for different parts of the sample. The experimental results are shown in figure 3. The dependences of these dimensions on the control parameter U (electric voltage) for different parts of the sample are qualitatively similar, but differ in magnitude. We attribute the quantitative difference to the strongly non-equilibrium distribution of the electric field along the sample. The field strength is markedly lower near the contacts (probes 1–1 and 3–3), compared with that in the middle of the sample (probes 2–2), which is characteristic of double-injection structures. The phase portraits obtained for these pairs of probes exhibit analogous behaviour. At $U_0 = 9.6$ V and $H = 4.5$ kOe we observed double cycles for the probes 1–1 and 3–3 and an attractor with a more complicated shape for the probes 2–2, i.e. several attractors occurred in the sample simultaneously. Such a loss of spatial coherence apparently indicates that the homogeneous semiconductor system disintegrates into subsystems with different numbers of degrees of freedom [14].

Contrary to the results of [15], chaotization of self-oscillations under an external periodic signal is observed solely in the case of a force-type action of the external perturbation. Parametric introduction of the external signal does not lead to chaotization in the system.

When the power of the introduced external periodic perturbation dissipated in the sample causes Joule heating, an additional parameter appears in the system, complicating

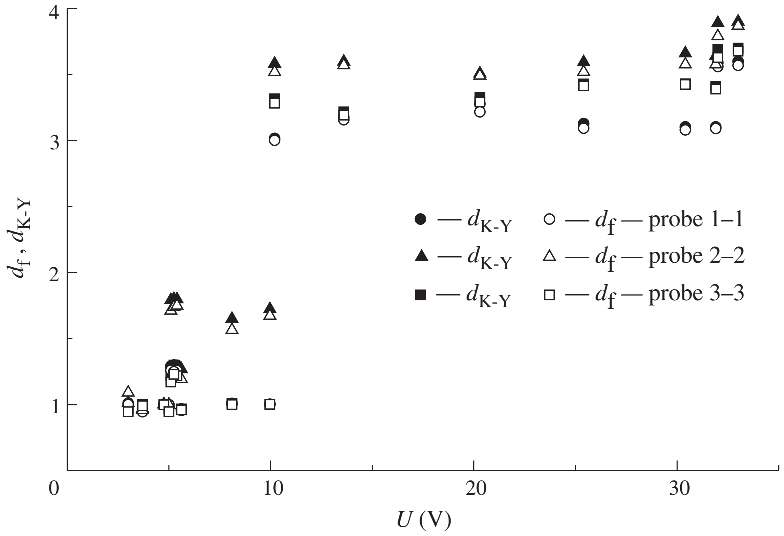

Figure 3. Dependences of fractal (d_f) and Kaplan–Yorke (d_{K-Y}) dimensions on voltage for different parts of the sample.

the interpretation of experimental data and impairing their reproducibility. The rise in the intensity of the fundamental harmonic with increasing control parameter E is in agreement with the well-known relation [4] for a screw instability

$$\omega = \frac{k}{2\pi}\mu_a E$$

where k is the wave-vector, μ_a is the ambipolar mobility and E is the electric field. It should be noted that, when approaching the frequency lock, the system in some cases demonstrates reverse bifurcations: frequency quadrupling, frequency doubling and limit cycle.

Raising the amplitude of the external signal results in basic changes. We carried out the following experiment: an external periodic perturbation was set, having fixed frequency and increasing amplitude, and its influence on the behaviour of the system in increasing electric field E was studied. It was found that, with the amplitude of the external signal increasing at small E, the amplitude of fundamental self-oscillations at frequency ω_0 goes to zero, i.e., frequency synchronization (lock) occurs, and the external signal is amplified. This mechanism of frequency lock, manifesting itself at sufficiently high amplitudes of the external perturbation, is usually referred to as 'synchronization by quenching' [5]. It should be noted that at small external signals, before the synchronization threshold, various types of beating arise in the sample. To these beats correspond various two-dimensional tori in the phase space, rebuilt depending on the frequency detuning. A resonance on a two-dimensional tori at frequency lock is typical of small periodic perturbation amplitudes, when a limit cycle with closed trajectory appears. The tori decay when the system passes to the synchronization mode and the limit cycle arises in the phase space, which is feature characteristic of large amplitudes of external perturbation, i.e. the case of synchronization by quenching. Different mechanisms of synchronization prevail, depending on the ratio of the external parameters: i.e. the electric field applied to the sample and the periodic perturbation.

2.2. Convective instability mode

For the screw instability, the convective mode is known to exist at before-threshold values of the parameters E and H, when the true criteria of absolute instability are not yet met [4]. The theory [5, 6] states that, if an external periodic signal is applied to a dynamic system on the threshold of a bifurcation (period doubling or Andronov–Hopf bifurcation), the signal with a frequency appearing upon the bifurcation is amplified. The condition for convective instability at $T = 77$ K is met at magnetic field strengths $H = 3.1$ and 6.1 kOe and applied pulsed electric voltages $V = 3.5$ and 2.2 V respectively. Let us consider the influence of the external periodic signal with fixed amplitude and widely varied frequency on the behaviour of the screw instability under a parametric perturbation. The sample is brought to the mode of absolute instability generation by selecting the electric field at a fixed magnetic field determining the threshold frequency. Then, the voltage is reduced until the oscillations disappear and a small periodic perturbation is applied to the pair of probes closest to the p^+ contact. The time series of the system are studied at a pair of probes lying closer to the n^+ contact. The output signal of the ac generator is maintained constant. Similarly to the case of absolute instability, when an external signal is applied to the system parametrically, a signal with the fundamental frequency ω_0 is generated in the sample, as indicated by the power spectrum, and there appears an external signal with even harmonics up to $8\omega_0$ and some other harmonics representing superpositions of these frequencies. In most cases, phase portraits demonstrate two-dimensional tori with trajectories closed or open, depending on the ratio of the fundamental harmonic frequencies, or with cycles having periods of up to $(6-8)\,\omega_0$. The amplification of the external signal has resonance nature (figure 4). The amount of amplification (up to 30 dB) depends not only on the external signal frequency, but also on the parametric space region (E, H) in which the system is studied. Of more interest are the data on amplification in the case when an external signal of certain frequency excites in the sample a natural threshold frequency and the ratio of these two frequencies is an integer (2, 3 and 4). The signal with the fundamental frequency ω_0 not only appears, but is also considerably amplified at applying an external signal $\omega_{ex} = \omega_0/2$, its magnitude exceeding 12–15-fold that of the external signal. Resonance amplification of the same order of magnitude is observed at $\omega_{ex} = \omega_0/4$ as well. In all of these cases, frequency lock occurs when the resonance is approached and a cycle with the corresponding period appears in the phase portrait instead of two-dimensional tori. Before the onset of synchronization, weak harmonics, multiples of both frequencies with a large denominator (up to 60), appear in the spectra, eventually ‘tuning’ the system through their participation in resonance phenomena.

With increasing frequency of the external signal ($\omega_{ex} > 2\omega_0$), the system response becomes simpler and usually follows the scheme of transitions: cycles with periods $1 \to 3 \to 2$ and an ordinary limit cycle due to the frequency introduced into the system.

2.3. Synchronization of chaotic oscillations

It should be noted that the synchronization of chaotic self-oscillations has threshold nature (i.e. there is a threshold amplitude of the external perturbation) and also depends on the multiplicity of the frequency introduced into the system. This can be accounted for by the fact that there exists a mechanism of local instability of motion in a system with a strange attractor. In addition, cases are observed in which the system transits to modes that are simpler in topological regard, when the Lyapunov dimension decreases owing to effects of self-organization in the system.

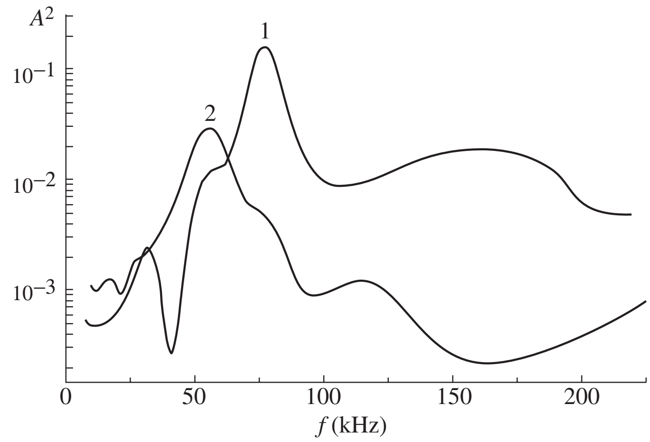

Figure 4. Power of the harmonic amplified to the greatest extent versus the frequency of an external periodic perturbation in different magnetic fields H: 3.1 kOe (curve 1) and 6.1 kOe (curve 2).

3. Conclusion

Study of the absolute and convective modes of screw instability excitation yielded the amplitudes and frequencies of the external periodic perturbation, for which motion regularization, signal amplification and frequency lock occur. The chaotic attractor observed in the system under external periodic action signal results in either a limit or a doubling cycle or in a torus with rational number of revolutions. The theory of nonlinear systems regards the synchronization under external periodic signal as a transition from a less ordered state to a more ordered one. From this point of view, the synchronization is a nonequilibrium phase transition. To classify nonequilibrium phase transitions, the beat frequency was chosen as the order parameter in our experiment. We believe that the synchronization by quenching is a first-order nonequilibrium phase transition, since the beat frequency shows a discontinuity in going across the boundary of the synchronization range and a hysteresis occurs, i.e. the system comes into, and out of the synchronization region at different magnitudes of frequency detuning. At the same time, frequency lock-in, characterized by a continuous change in the order parameter and absence of hysteresis, is a second-order phase transition.

Acknowledgments

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